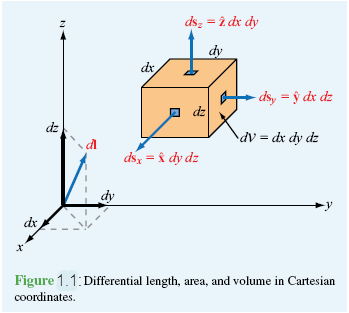
**Chapter-5**

**Coordinate systems**

**Cartesian coordinates:**

****

**Figure:** Differential length, area, and volume in Cartesian coordinates.

Cartesian variables 

Vector representation, 

Magnitude of  is , 

Position vector, for 

Base vector properties:  and 



Dot product 

Cross product 

Differential length,

Differential surface area,

Differential volume,

**Cylindrical coordinates:**

|  |  |
| --- | --- |
|  |  |
| **Figure:** Differential length, area, and volume in cylindrical coordinates. | **Figure:** Point in cylindrical coordinates; is the radial distance from the origin in the plane, is the angle in plane measured from the axis toward the axis, and is the vertical distance from the plane. |

Cylindrical variables 

Vector representation, 

Magnitude of  is , 

Position vector, for 

Base vector properties:  and 

Dot product 

Cross product

Differential length,

Differential surface area,

Differential volume,

**Spherical coordinates:**

|  |  |
| --- | --- |
|  |  |
| **Figure:** point in spherical coordinates. | **Figure:** Differential volume in spherical coordinates. |

Spherical variables 

Vector representation, 

Magnitude of  is , 

Position vector, for 

Base vector properties:  and 



Dot product 

Cross product 

Differential length,

Differential surface area,

Differential volume,

**Transformation (Cartesian coordinates to cylindrical and vice versa):**

|  |  |
| --- | --- |
|  |  |
| **Figure:** Interrelationships between Cartesian coordinates and cylindrical coordinates. | **Figure:** Interrelationships between base vectors and . |

We know, 

 ; 

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  | 0 |
|  |  |  |  | 0 |
|  |  | 0 | 0 | 1 |

That is, 



**Transformation (Spherical coordinates to cylindrical coordinates and vice versa):**

We know, 

 ; 

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  | 0 |
|  |  |  |  | 0 |
|  |  | 0 | 0 | 1 |

That is, 



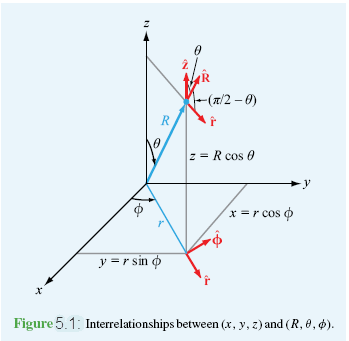
**Transformation (spherical coordinates to Cartesian coordinates and vice versa):**

We know,  ;

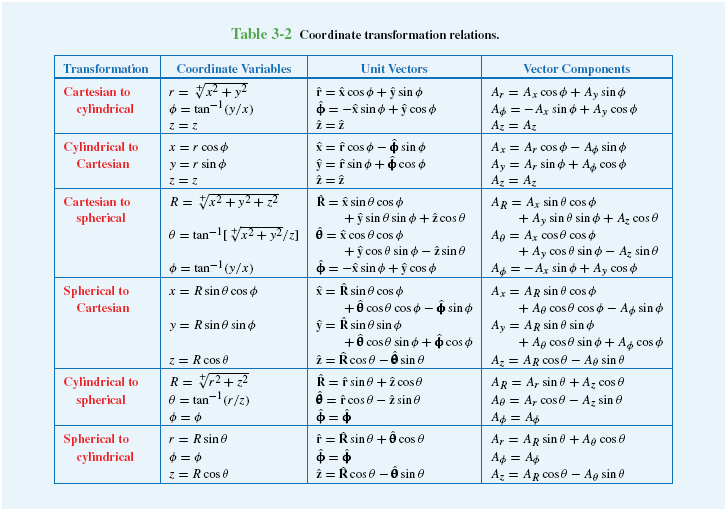


|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  | 0 |

That is,



**Figure:** Interrelationships between and

****

**Example 1:** Transform from cylindrical coordinates to Cartesian coordinates.

**Solution:** ,

,

So, Cartesian point

**Example 2:** Transform from Cartesian coordinate to cylindrical coordinate.

**Solution:**

**Example 3:** Transform from Cartesian coordinate to spherical coordinate.

**Solution:**

So, the spherical point

**Example 4:** Transform from spherical coordinate to Cartesian coordinate.

**Solution:**

So, the Cartesian point

**Example 5:** Transform from spherical coordinate to cylindrical coordinate.

**Solution:**

So, the cylindrical pint

**Example 6:** Transform from cylindrical coordinate to spherical coordinate.

**Solution:**

So, the spherical point

**Example 7:** Express vector in spherical coordinate.

**Solution:** We know in spherical coordinate

**Example 8:** Expressvector in Cartesian form.

**Solution:** We know in Cartesian coordinate

**Example 9:** Transform vector to cylindrical coordinates.

**Solution:** We know, cylindrical coordinate

**Example 10:** Express vector in Cartesian coordinate.

**Solution:**  We know in Cartesian coordinate

Hence,

**Example 11:** Transform the vector to spherical coordinate.

**Solution:** We know in spherical coordinate

**Example 12:** Transform the vector  at the point  to cylindrical coordinates.

**Solution:** Given that,  at the point .

Here, 

We get, 

Now, from given equation

We know,



Hence, 

**Example 13:** Transform the vector . to spherical coordinates.

**Solution:** Given that , 

Here, 

We get, 



We know, 

Here, 





At the point  .

**Sample exercise-5.1**

1. **Convert the coordinates of the following points from Cartesian to cylindrical and spherical coordinates:**
2. **Ans:** and
3. **Ans:** and
4. **Ans:** and
5. **Convert the coordinates of the following points from cylindrical to Cartesian and spherical coordinates:**
6. **Ans:** and
7. **Ans:** and
8. **Ans:** and
9. **Convert the coordinates of the following points from spherical to Cartesian and cylindrical coordinates:**
10. **Ans:** and
11. **Ans:** and
12. **Ans:** and
13. **Transform the following vectors into cylindrical coordinates at the indicated points:**
14. **Ans:**
15. **Ans:**
16. **Transform the following vectors into spherical coordinates at the indicated points:**
17. **Ans:** .
18. **Ans:**
19. **Transform the following vectors into cartesian coordinates at the indicated points:**
20. **Ans:**
21. **Ans:**